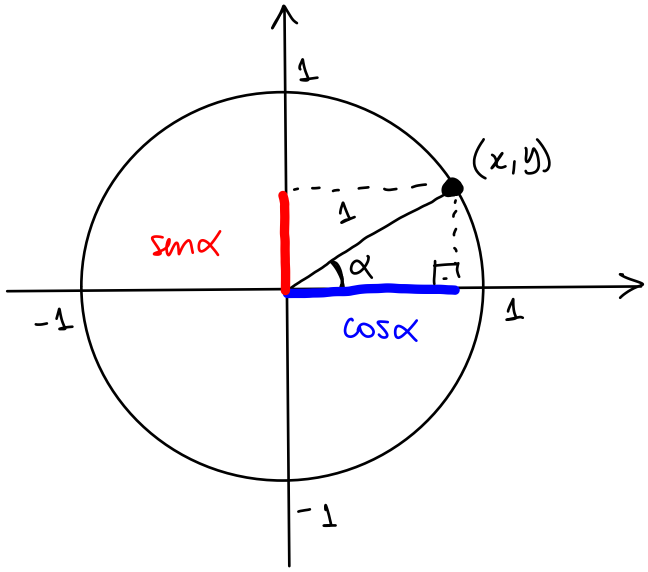
Exercício 5. A diferença entre o maior e o menor valor da função $g(x) = 4 + 2\,\text{sen}\left(3x + \dfrac{\pi}{4}\right)$, definida em $\mathbb{R}$, é:



$$\cos\alpha = \frac{x}{1} \qquad \text{sen}\,\alpha = \frac{y}{1}$$

$$-1 \leq \cos\alpha \leq 1$$

$$-1 \leq \text{sen}\,\alpha \leq 1 \leftarrow$$

$$-1 \leq \text{sen}\left(3x + \frac{\pi}{4}\right) \leq 1 \overset{(\times 2)}{\Rightarrow} -2 \leq 2\,\text{sen}\left(3x + \frac{\pi}{4}\right) \leq 2$$

$$\overset{(+4)}{\Rightarrow} 2 = 4 - 2 \leq \underbrace{4 + 2\,\text{sen}\left(3x + \frac{\pi}{4}\right)}_{g(x)} \leq 4 + 2 = 6$$

$\therefore\ 2 \leq g(x) \leq 6$

Diferença: $6 - 2 = 4$.

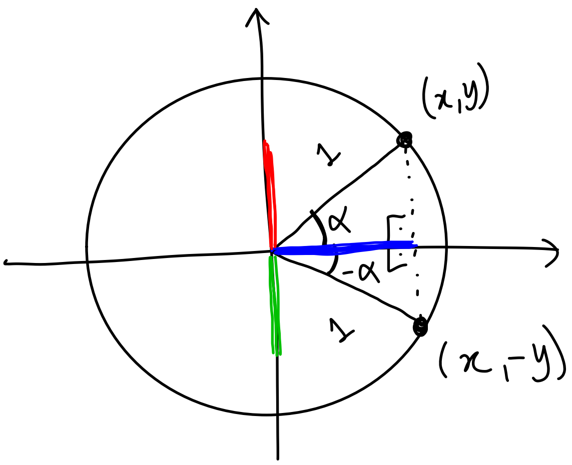
$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \operatorname{sen}\alpha \cdot \operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha \cdot \cos\beta + \operatorname{sen}\beta \cdot \cos\alpha$$

$$\cos(-\alpha) = \cos\alpha \qquad (\text{par})$$

$$\operatorname{sen}(-\alpha) = -\operatorname{sen}\alpha \qquad (\text{ímpar})$$



$$x = \cos\alpha = \cos(-\alpha)$$

$$-y = -\operatorname{sen}\alpha = \operatorname{sen}(-\alpha)$$

$$\cos(\alpha-\beta) = \cos(\alpha+(-\beta)) = \cos\alpha \cdot \cos(-\beta) - \operatorname{sen}\alpha \cdot \operatorname{sen}(-\beta)$$

$$= \cos\alpha \cdot \cos\beta + \operatorname{sen}\alpha \cdot \operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}(\alpha+(-\beta)) = \operatorname{sen}\alpha \cdot \cos(-\beta) + \operatorname{sen}(-\beta) \cdot \cos\alpha$$

$$= \operatorname{sen}\alpha \cdot \cos\beta - \operatorname{sen}\beta \cdot \cos\alpha$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \ldots$$

$$\operatorname{tg}(a+b) = \frac{\operatorname{tg}a + \operatorname{tg}b}{1 - \operatorname{tg}a \times \operatorname{tg}b}$$

$$\operatorname{tg}(a-b) = \frac{\operatorname{tg}a - \operatorname{tg}b}{1 + \operatorname{tg}a \times \operatorname{tg}b}$$

Exercício 14. Se $g : \mathbb{R} \longrightarrow \mathbb{R}$ é a função definida por $g(x) = 3x + \text{sen}\left(\frac{\pi}{2}x\right)$. Então o valor da soma $g(2) + g(3) + ... + g(11)$ é:

a) 183.

b) 187.

c) 190.

~~d)~~ 194.

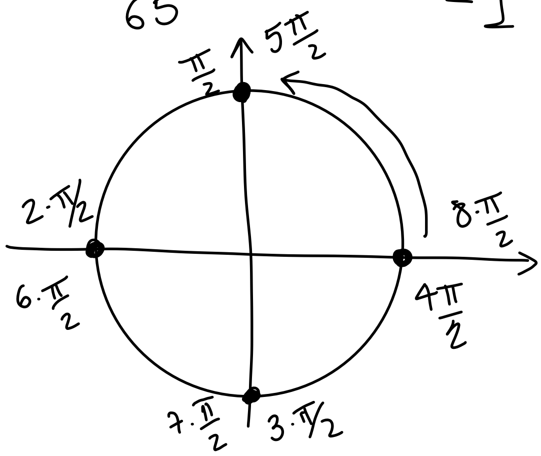
$$S = g(2) + g(3) + \cdots + g(11)$$

$$= 3\cdot 2 + \text{sen}\left(\frac{\pi}{2}\cdot 2\right) + 3\cdot 3 + \text{sen}\left(\frac{\pi}{2}\cdot 3\right) + \cdots$$

$$+ 3\cdot 11 + \text{sen}\left(\frac{\pi}{2}\cdot 11\right)$$

$$= 3\cdot 2 + 3\cdot 3 + \cdots + 3\cdot 11 + \text{sen}\left(\frac{\pi}{2}\cdot 2\right) + \text{sen}\left(\frac{\pi}{2}\cdot 3\right) + \cdots + \text{sen}\left(\frac{\pi}{2}\cdot 11\right)$$

$$= 3\underbrace{(2+3+\cdots+11)}_{65} + \underbrace{\cancel{\text{sen}\left(\frac{\pi}{2}\cdot 3\right)}}_{=-1} + \cancel{\text{sen}\left(\frac{\pi}{2}\cdot 5\right)} + \cancel{\text{sen}\left(\frac{\pi}{2}\cdot 7\right)} + \cancel{\text{sen}\left(\frac{\pi}{2}\cdot 9\right)} + \underbrace{\text{sen}\left(\frac{\pi}{2}\cdot 11\right)}_{=-1}$$



$$\text{sen}\left(2k\cdot\frac{\pi}{2}\right) = 0$$

$$\text{sen}\left((2k+1)\frac{\pi}{2}\right) = 1 \text{ ou } -1$$

$$\cancel{1+}2 + \cdots + n = \frac{n(n+1)}{2} - 1 \quad \therefore \quad n = 11 : \frac{11(11+1)}{2} - 1$$

$$= 65$$

Portanto,

$$S = 3\cdot 65 - 1 = 194$$